DECLINE CURVES

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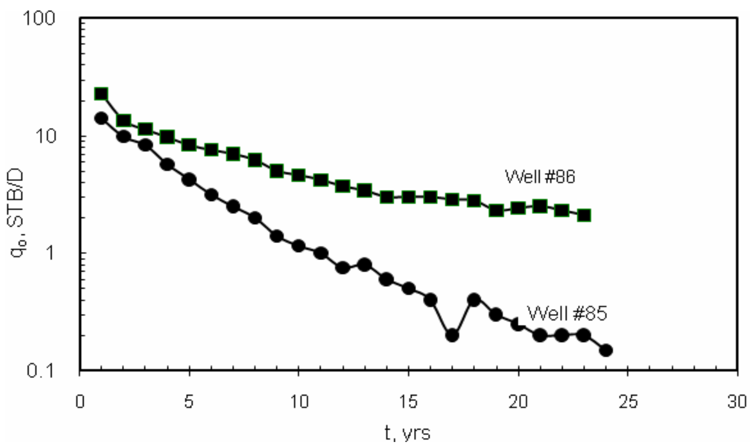
Oil and gas production rates decline as a function of time. Loss of reservoir pressure or the changing relative volumes of the produced fluids are usually the cause. Fitting a line through the through the performance history and assuming this same line trends similarly into the future forms the basis for the decline curve analysis concept.

The following figure shows semilog rate – time decline curves for two different well located in the same field. Note the logarithmic scale for the rate side.



HISTORICAL PERSPECTIVE

- Arps (1945) (1956) collected these ideas into a comprehensive set of line equations defining exponential, hyperbolic and harmonic curves.
- Brons (1963) and Fetkovich (1983) applied the constant pressure solution to the diffusivity equation to show that the exponential decline curve actually reflects single phase, incompressible fluid production from a closed reservoir. In other words its meaning was more than just an empirical curve fit.
- Fetkovich (1980) (1983) developed a comprehensive set of type curves to enhance the application of decline curve analysis.
- The advent of the personal computer revolutionized the analysis of decline curves by making the process less time consuming. .
- Doublet and Blasingame (1995) developed the theoretical basis for combining transient and boundary dominated production behavior for the pressure transient solution to the diffusivity equation.

A production history may vary from a straight line to a concave upward curve. In any case the object of decline curve analysis is to model the production history with the equation of a line. The following table summarizes the five approaches for using the equation of a line to forecast production.

Log Rate-Time Shape	Name	Model	Decline
Straight	Exponential		Stepwise
Straight	Exponential	Arps	Continuous straight
Curved but converging	Hyperbolic	Arps	Continuous curve
Curved but limit	Harmonic	Arps	Continuous curve which nearly converges
Curved but not converging	Amended		Dual – Infinite acting amended to a limiting curve

Arps applied the equation of a hyperbola to define three general equations to model production declines. These models are; exponential, hyperbolic and harmonic. In order to locate a hyperbola in space one must know the following three variables. The starting point on the “y” axis. (q_i), initial rate. (D_i).the initial decline rate, the degree of curvature of the line (b).

EXPONENTIAL DECLINE - There are two basic definitions for expressing the exponential decline rate.

- Effective or constant percentage decline expresses the incremental rate loss concept in mathematical terms as a stepwise function.
- Nominal or continuous rate decline expresses the negative slope of the curve representing the hydrocarbon production rate versus time for an oil gas reservoir.

The accompanying equation shows the relationship between nominal and effective, decline rates. $D = -\ln(1 - d)$
Convention assumes the decline rate is expressed in terms of (%/yr). Comparison of rate, time and cumulative production relationships for both definitions are shown in the following table.

Constant Percentage (Effective) and Continuous (Nominal) Exponential Equations

	Constant Percentage	Continuous
Decline rate	$d = \frac{q_1 - q_2}{q_1}$	$D = \frac{\ln\left(\frac{q_1}{q_2}\right)}{t}$
Producing rate	$q_2 = q_1(1-d)^t$	$q_2 = q_1 \exp(-Dt)$
Elapsed time	$t = \frac{\ln\left(\frac{q_2}{q_1}\right)}{-\ln(1-d)}$	$t = \frac{\ln\left(\frac{q_1}{q_2}\right)}{D}$
Cumulative recovery	$Q_p = \frac{q_1 - q_2}{-\ln(1-d)}$	$Q_p = \frac{q_1 - q_2}{D}$

THE ARPS EQUATIONS - The following discussion applies the previously developed general equations to the Arps definitions for exponential, hyperbolic and the special case harmonic production decline curves. Arps defined the following three cases.

(b = 0) for the exponential case,
(0<b<1) for the hyperbolic case, and,
(b = 1) for the harmonic case.

The following table summarizes these rate, time, cummulative production and decline rate equations.

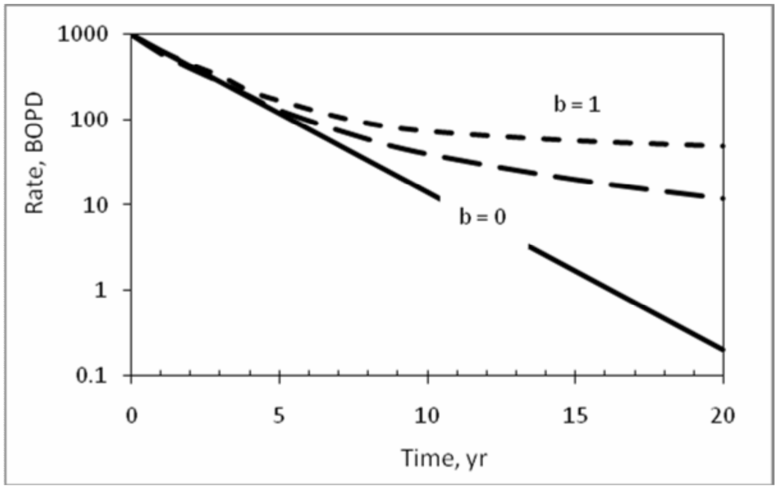
The Arps Equations

	$b = 0$	$0 < b < 1$	$b = 1$
D	$\frac{\ln\left(q_1 \square q_2\right)}{t}$	$D_1\left(\frac{q_2}{q_1}\right)^{\frac{1}{b}}$	$D_1\frac{q_2}{q_1}$
q	$q_1\exp(-Dt)$	$\frac{q_1}{(1+btD_1)^{\frac{1}{b}}}$	$\frac{q_1}{(1+bD_1)}$
Q_p	$\frac{q_1 - q_2}{D}$	$\frac{q_1}{D_1(1-b)}\left[1-\frac{q_2}{q_1}\right]^{1-b}$	$\frac{q_1}{D_1}\ln(1 + D_1 t)$
t	$\frac{\ln\left(q_1 \square q_2\right)}{D}$	$\frac{\left(\frac{q_1}{q_2}\right)^{b} - 1}{bD_1}$	$\frac{q_1 - q_2}{D_1 q_2}$

CURVE CHARACTERISTICS

- All rate-time curves must trend in a downward manner.
- The semilog rate-time curve is a straight line for the exponential equation while the hyperbolic and harmonic decline lines are curved
- The Cartesian rate-cumulative recovery plots are a straight line for the exponential case, while the hyperbolic and harmonic lines are curved.
- A semilog rate-cumulative production plot for the harmonic equation results in a straight line while the exponential and hyperbolic declines are curved.

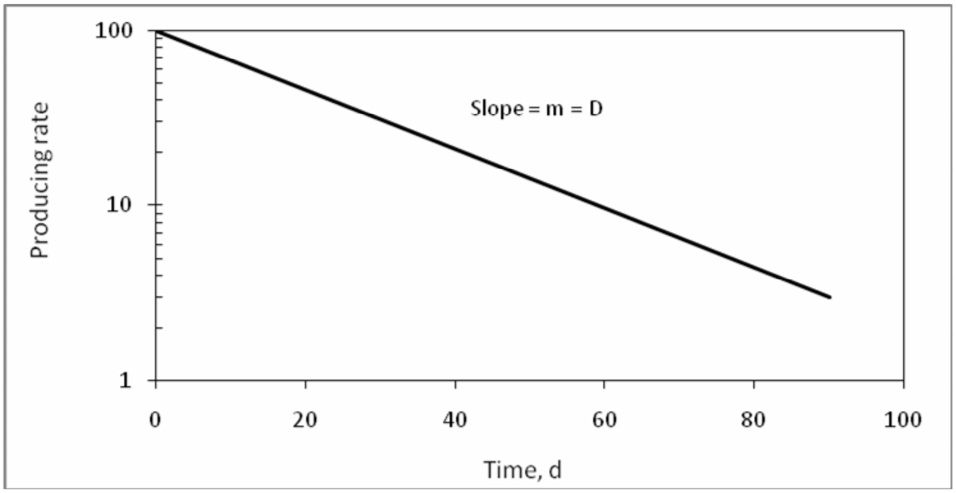
The following figure presents the general semilog rate-time plot for the Arps exponential, hyperbolic and harmonic equations. Note how the harmonic curve tends to flatten out with time.



BOUNDS OF THE ARPS EQUATIONS - Theoretically, the b-exponent term included in the rate-time equation could vary in a positive or negative manner. A negative b-exponent value implies an increasing production rate indicates production extends to infinity, hence cumulative production must be infinite for the ($b \geq 1$) cases. This statement shows why the b-exponent term cannot be greater than unity.

These studies indicate the decline exponent must vary over the ($\underline{0} < b < 1$) range to apply the Arps curves in a practical sense. The harmonic case should be used only with reservations because a forward prediction would result in an infinite cumulative recovery estimate.

THE CONSTANT PRESSURE SOLUTION - Fetkovich expressed the Van Everdingen-Hurst constant pressure solution to the diffusivity equation for a closed, circular reservoir in the form of an exponential equation. A straight line may be constructed from the solution. The following figure is a typical figure of the semi-log rate-time plot which is exactly similar to the Arps definition. The solution indicates the exponential decline curve is the result of a known set of reservoir conditions.



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AUTHOR

Dr. Steven W. Poston is a retired Petroleum Engineer and Texas A&M Professor Emeritus with an extensive background in the industry and academia. He holds a B.S. degree in Geological Engineering and a PhD. in Petroleum Engineering from Texas A&M University. Dr. Poston began his career with Gulf Oil in 1967, and worked in various engineering and supervisory roles during 13 years there. He then served as a Professor at Texas A&M, teaching graduate and undergraduate courses there before retiring in the mid 1990's.

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